

Anisotropic Extra Dimensions

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We consider the scenario where in a five-dimensional theory, the extra spatial dimension has different scaling than the other four dimensions. We find background maximally symmetric solutions, when the bulk is filled with a cosmological constant and at the same time it has a three-brane embedded in it. These background solutions are reminiscent of Randall-Sundrum warped metrics, with bulk curvature depending on the parameters of the breaking of diffeomorphism invariance. Subsequently, we consider the scalar perturbation sector of the theory and show that it has certain pathologies and the striking feature that in the limit where the diffeomorphism invariance is restored, there remain ghost scalar mode(s) in the spectrum.

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I. INTRODUCTION

Theories with extra spacetime dimensions were introduced in the '20s by the work of Kaluza and Klein (KK) and gained further attention with the advance of string theory, where they were a crucial ingredient for the self-consistency of the theory. During the last decade, the interest in model building and phenomenology of extra dimensional theories has been renewed with the discovery that they can play an important role in physics of energies/distances which are probed in current experiments and observations [1]. There have been a wealth of models where extra dimensions open up at the electroweak scale and provide new insights of standard high energy problems [2], or at macroscopic scales and modify gravity in the infrared [3].

A common factor of these models is that the starting point in model building is a higher dimensional diffeomorphism invariant theory. The background solutions of the metric, *i.e.* the gravitational vacuum of the theory, spontaneously breaks the higher-dimensional diffeomorphism group down to its four-dimensional subgroup. This constitutes a gravitational Higgs mechanism and provides masses for the towers of KK gravitons, vectors and scalars [4–6].

Recently, Hořava proposed a theory of gravity which also breaks diffeomorphism invariance [7]. The new theory is supposed to be an adequate ultraviolet (UV) completion of General Relativity above the Planck scale. Its basic assumption is the existence of a preferred foliation by three-dimensional constant time hypersurfaces, which splits spacetime into space and time. This allows to add higher order spatial derivatives of the metric to the action, without introducing higher order time derivatives. This is supposed to improve the UV behaviour of the graviton propagator and render the theory power-counting renormalisable without introducing ghost modes, which are common when adding higher order curvature invariants to the action in a covariant manner [8].

Such a theory cannot be invariant under the full set of diffeomorphisms, but it can still be invariant under the more limiting foliation-preserving diffeomorphisms. This breaking of the full diffeomorphisms group down to its foliation-preserving subgroup is, however, explicit and not spontaneous. This has been the source of problems, related with a scalar degree of freedom, whose behaviour creates a serious viability issue for all versions of the theory. In the version with “projectability” and for maximally symmetric backgrounds, the scalar is either (classically) unstable or it becomes a ghost (quantum-mechanically unstable) [9–12]¹. Also the non-projectable version of the theory has strong coupling problems and instabilities [14–16].

In the present paper, we will revisit the extra dimensional theories and break explicitly, *i.e.* at the action level, the higher dimensional diffeomorphism invariance to its foliation-preserving subgroup. The foliation that we will choose is adapted to an extra space dimension, as opposed to Hořava-type of theories where it was adapted to the time dimension. This will leave intact the four-dimensional spacetime diffeomorphisms contrary to Hořava theory. It is evident that in the resulting theory we can not demand the theory to be renormalisable, as it was the motivation in the Hořava theory, since that would indicate the inclusion of higher order *four-dimensional curvature* invariants, thus introducing ghosts. On the other hand, depending on the difference in scaling of the extra dimension compared with the scaling of the four ordinary spacetime dimensions, higher powers of the extrinsic curvature of the four dimensional hypersurfaces may play part in the effective theory, before reaching energies where inevitably ghost-bearing higher order dimensional operators appear.

¹ See however [13] and references therein for conditions under which the classical instability does not show up.

The aim of this paper is to study such a theory where diffeomorphism invariance in an extra dimensional model is explicitly broken. We will pursue this aim in a theory, which in the limit of diffeomorphism invariance restoration, tends to the Randall Sundrum warped model with one brane and infinite extra dimension. We will formulate the theory, find background solutions and study the scalar sector perturbations around this background. This study will reveal that such an explicit breaking of the five-dimensional diffeomorphism invariance is dangerous and introduces ghost scalar mode(s).

The pathological behaviour of gravity at small distances is a problem that plunges many gravity theories. In the direction of proposing a UV-completion theory of General Relativity, higher order curvature terms were introduced. The so-called modified gravity theory, includes $f(R)$ gravity models [17], and the Gauss-Bonnet (GB) models [18]. The $f(R)$ models have a better behaviour in the UV but they have serious cosmological problems [19] (see however [20]). For GB models it was shown that tensor perturbations are typically plagued by instabilities in the UV.

Similar problems arise by considering modifications of gravity at large distances. The Dvali-Gabadadze-Porrati (DGP) braneworld model [3] and its extension [21] with a GB term in the bulk, was proposed as an alternative to the acceleration of the Universe without the need of dark energy [22, 23]. However, it was soon realized that the DGP model contains a ghost mode [24], which casts doubts on the viability of the self-accelerating solution. Other theories that modify gravity at large distances were proposed by introducing a mass term to the graviton via a potential term. This was explored many years ago by Fierz and Pauli [25]. Later it was realized that gravitational theories with a mass term are behaving quite unlike other theories and are typically accompanied by several problems, most notably ghost/tachyon instabilities and strong coupling issues. Also massive gravity theories have a very characteristic feature the Van Dam-Veltman-Zakharov (vDVZ) discontinuity [26, 27], *i.e.* the fact that as the mass of the graviton goes to zero the scalar graviton mode fails to decouple (see, however, a way out in curved space [28, 29]). To cure this problem one has to advocate nonlinear dynamics, employing the Vainshtein mechanism [30], but then a ghost appears in the spectrum first noticed by Boulware and Deser [31].

A similar behaviour like the vDVZ discontinuity in massive gravity is observed in our theory: although the background solutions tend to the Randall-Sundrum solutions as the diffeomorphism invariance is restored, the problematic scalar modes are not removed from the spectrum in the same limit. We attribute this behaviour to the explicit breaking of the diffeomorphism invariance in the theory.

II. BREAKING THE DIFFEOMORPHISM INVARIANCE

Our starting point is five-dimensional Einstein gravity in the presence of a cosmological constant and a brane embedded at some point $y = 0$ of the extra dimension. We assume Z_2 symmetry along the y -direction with $y = 0$ as the fixed point. The corresponding action is given by

$$S = \int d^5x \left(R^{(5)} - 2\Lambda_5 \right) \sqrt{-G} - \int_{brane} d^4x \sqrt{-g} \sigma, \quad (2.1)$$

where $R^{(5)}$ is the five-dimensional Einstein-Hilbert term and Λ_5 , σ and G_{MN} is the five-dimensional cosmological constant, the brane tension and the metric respectively. In the following we wish to modify the above theory in a way that diffeomorphism invariance is broken along the extra dimension.

First, let us make the following ADM splitting along the extra dimension y

$$ds^2 = dy^2 N^2 c^2 + g_{\mu\nu} (N^\mu dy + dx^\mu) (N^\nu dy + dx^\nu), \quad (2.2)$$

with $g_{\mu\nu}$ the four-dimensional metric. In the modification that we will discuss, we shall adopt an anisotropic scaling of the different dimensions with $[x^\mu] = -1$ and $[y] = -w$, where the extra dimension singles out. For such a theory, the ADM metric components scale as $[g_{\mu\nu}] = 0$, $[N] = 0$ and $[N^\nu] = w - 1$. The five-dimensional action (2.1) can be generalised as

$$S = \int d^4x dy N \sqrt{-g} \left[\frac{\rho}{2} \left(R^{(4)} - 2\Lambda_5 \right) - \frac{2}{\kappa^2} (K_{\mu\nu} K^{\mu\nu} - \lambda K^2) \right] - \int_{brane} d^4x \sqrt{-g} \sigma, \quad (2.3)$$

where the extrinsic curvature $K_{\mu\nu}$ is given by $[K_{\mu\nu}] = w$, with

$$K_{\mu\nu} = \frac{1}{2N} (\partial_y g_{\mu\nu} - \nabla_\mu N_\nu - \nabla_\nu N_\mu), \quad (2.4)$$

and the scaling of the various quantities are $[\rho] = w + 2$, $[\kappa] = \frac{w-4}{2}$, $[R^{(4)}] = [\Lambda_5] = 2$. The above action receives radiative corrections and can be extended by including higher dimensional operators. Depending on the value of w ,

there can be higher powers of the extrinsic curvature important at energies lower than the one that unitarity of the theory is lost (typically when R^2 terms dominate). We will discuss about that possibility in a later section. For the moment we will restrict ourselves to the classical generalised action (2.3).

The parameter λ is crucial for the following discussion. It represents the breaking of the five-dimensional diffeomorphism invariance to its four dimensional subgroup. The restricted symmetry allows for a different weighting of the $K_{\mu\nu}^2$ and K^2 terms in the action, contrary to the fully covariant theory.

The field equations coming from variation of (2.3) with respect to the fields N, N_μ and $g_{\mu\nu}$ are given by

$$0 = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta N} = \frac{\rho}{2} \left(R^{(4)} - 2\Lambda_5 \right) + \frac{2}{\kappa^2} (K_{\mu\nu} K^{\mu\nu} - \lambda K^2) , \quad (2.5)$$

$$0 = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta N_\mu} = -\frac{4}{\kappa^2} \nabla_\mu \pi^{\mu\nu} , \quad (2.6)$$

$$0 = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} = -\frac{\rho}{2} \left(G_{(4)}^{\mu\nu} + g^{\mu\nu} \Lambda_5 \right) N \\ + \frac{2}{\kappa^2} \left[\partial_y \pi^{\mu\nu} + N K \pi^{\mu\nu} + 2 \nabla_\sigma \left(\pi^{\sigma(\mu} N^{\nu)} \right) - N_\kappa \nabla^\kappa \pi^{\mu\nu} + 2 N K^{\sigma\mu} \pi^\nu_{\sigma} \right] \\ - \frac{1}{2} g^{\mu\nu} \frac{2}{\kappa^2} (K_{\mu\nu} K^{\mu\nu} - \lambda K^2) - \frac{1}{2} \sigma \delta(y) g^{\mu\nu} , \quad (2.7)$$

where the y -canonical momentum of the four-dimensional metric is

$$\pi^{\mu\nu} = K^{\mu\nu} - \lambda K g^{\mu\nu} . \quad (2.8)$$

As in the original case of Randall-Sundrum, it is important to consider the junction conditions of the system. These junction conditions involving the brane tension, are found by identifying the distributional terms in the equations of motion. It is fairly easy to see from (2.7), that the only distributional term is $\partial_y \pi^{\mu\nu}$. Integrating along the extra dimension and taking the limit close to the brane, gives the junction conditions, which takes the following familiar form

$$\frac{2}{\kappa^2} [\pi^{\mu\nu}]_-^+ = \frac{1}{2} g^{\mu\nu} \sigma . \quad (2.9)$$

Under the Z_2 symmetry the above relation can be rewritten as

$$\frac{4}{\kappa^2} \pi^{\mu\nu} = \frac{1}{2} g^{\mu\nu} \sigma . \quad (2.10)$$

In the following section, we will present background solutions for this theory which possess maximal symmetry in four dimensions. We will see that these solutions bear great similarity with the Randall-Sundrum solutions [2] and their curved version [32], with λ -dependent bulk curvatures.

III. SOLUTIONS WITH MAXIMALLY SYMMETRIC BACKGROUNDS

In this section we will consider solutions of the field equations where the four-dimensional metric $g_{\mu\nu}$ is maximally symmetric. This metric evaluated at the brane position, is also the induced metric on the brane at $y = 0$, since the brane is static. Therefore, the solutions for the bulk metric, satisfying the appropriate boundary conditions, define families of maximally symmetric brane solutions.

A. Flat Brane

Let us first examine the case where $g_{\mu\nu}$ is flat. In this case we will consider the following ansatz for the metric components

$$N_\mu = 0, \quad g_{\mu\nu} = e^{2f(y)} \eta_{\mu\nu}, \quad N = 1 , \quad (3.1)$$

allowing for a warp factor $f(y)$ along the extra dimension. Then, eq. (2.5) becomes

$$\kappa\Lambda_5\rho + \frac{8(-1+4\lambda)(f'(y))^2}{\kappa} = 0, \quad (3.2)$$

and eq. (2.6) is automatically satisfied while eq. (2.7) yields

$$\kappa^2\Lambda_5\rho + \kappa^2\sigma\delta(y) + 8(-1+4\lambda)(f'(y))^2 + 4(-1+4\lambda)f''(y) = 0. \quad (3.3)$$

Depending on the value of the parameter λ we distinguish the three following cases of solutions that we need to examine.

1. Case 1 ($\lambda < \frac{1}{4}$)

In this case, (3.2) gives

$$f(y) = -\frac{|y|\kappa}{2\sqrt{2}}\sqrt{\frac{\Lambda_5\rho}{1-4\lambda}}. \quad (3.4)$$

Clearly, $\rho\Lambda_5$ must be positive. Substituting the above solution to (3.3) we see that it is trivially satisfied.

2. Case 2 ($\lambda > \frac{1}{4}$)

In this case, we have the same form of solution, but now $\rho\Lambda_5$ is negative. Again, eq. (3.3) is satisfied. For $\lambda = 1$ we get a Randall-Sundrum-like solution [2].

3. Case 3 ($\lambda = \frac{1}{4}$)

For this case, we get that $\Lambda_5 = 0$ and $f(y)$ is arbitrary. This is a critical point of the theory, where the bulk theory is conformally invariant.

Substituting $g_{\mu\nu}$ in (2.10) we get

$$\kappa^2\sigma + 8(-1+4\lambda)f'(y) = 0. \quad (3.5)$$

This equation can also be reproduced by integrating the distributional parts of (3.3). Substituting $f(y)$ gives the following expression for the tension

$$\sigma = -2\sqrt{2}\frac{(1-4\lambda)}{\kappa}\sqrt{\frac{\Lambda_5\rho}{1-4\lambda}}. \quad (3.6)$$

Note that in the case, where the bulk is conformally invariant, the brane is tensionless. Furthermore in the limit where $\lambda \rightarrow 1$, we get the familiar result of a positive tension brane.

B. Curved Brane

Instead of using a flat $g_{\mu\nu}$, we can introduce a metric of constant non-zero curvature. Namely, we will have

$$ds_{(5)}^2 = ds_{(4)}^2 + dy^2, \quad (3.7)$$

where

$$ds_{(4)}^2 = \alpha(y)^2(-dt^2 + e^{2Ht}\delta_{ij}dx^i dx^j) \quad (3.8)$$

is a conformally flat, Einstein space of constant non-zero curvature [33]. Again, eq. (2.6) is automatically satisfied. For the other two equations (2.5) and (2.7), we have respectively

$$-\Lambda_5 \rho + \frac{1}{\kappa^2 \alpha(y)^2} \left(6H^2 \kappa^2 \rho + (8 - 32\lambda) (\alpha'(y))^2 \right) = 0 , \quad (3.9)$$

and

$$\frac{3H^2 \rho}{2\alpha(y)^4} - \frac{\Lambda_5 \rho}{2\alpha(y)^2} - \frac{\sigma}{2\alpha(y)^2} \delta(y) + \frac{2(1-4\lambda)}{\kappa^2 \alpha(y)^4} \left((\alpha'(y))^2 + \alpha(y) \alpha''(y) \right) = 0 . \quad (3.10)$$

Solving (3.9) for H^2 and substituting in (3.10) we obtain

$$\kappa^2 \alpha(y) (\Lambda_5 \rho + 2\sigma \delta(y)) + 8(-1 + 4\lambda) \alpha''(y) = 0 . \quad (3.11)$$

This is an equation for the warp factor $a(y)$ and its solution will depend on the diffeomorphism breaking parameter λ . It is useful to see that eqs. (3.9) and (3.11) in the limit where $\lambda \rightarrow 1$ match with equations (6), (7) of [33] with $\kappa^2 \rightarrow \frac{1}{M^3}$, $\rho \rightarrow 4M^3$ and $\Lambda_5 \rightarrow \Lambda_5/4M^3$.

Taking into account (2.10), we obtain as before three cases:

$$1. \quad \underline{\text{Case 1 } (\lambda < \frac{1}{4})}$$

In the case where $\lambda < \frac{1}{4}$, the solution is

$$\alpha(y) = \cos(my) - \frac{\sigma m}{\rho \Lambda_5} \sin(m|y|) , \quad (3.12)$$

with

$$m^2 = -\frac{\kappa^2 \rho}{8|1-4\lambda|} \Lambda_5 , \quad (3.13)$$

Λ_5 is negative and

$$H^2 = \frac{1}{48} \left(8\Lambda_5 - \frac{\kappa^2 \sigma^2}{\rho(1-4\lambda)} \right) = \frac{1}{6\Lambda_5 \rho^2} (\sigma^2 m^2 + \Lambda_5^2 \rho^2) . \quad (3.14)$$

$$2. \quad \underline{\text{Case 2 } (\lambda = \frac{1}{4})}$$

In the conformal point $\Lambda_5 = 0$, $H^2 = 0$ and the brane is tensionless ($\sigma = 0$).

$$3. \quad \underline{\text{Case 1 } (\lambda > \frac{1}{4})}$$

Finally for the case where $\lambda > \frac{1}{4}$ we have the following solution

$$\alpha(y) = \cosh(my) + \frac{\sigma m}{\rho \Lambda_5} \sinh(m|y|) , \quad (3.15)$$

with

$$m^2 = -\frac{\kappa^2 \rho}{8|1-4\lambda|} \Lambda_5 . \quad (3.16)$$

In the above relation, Λ_5 is negative and furthermore

$$H^2 = \frac{4|1-4\lambda|}{3\rho^3 \kappa^2} \frac{m^2}{\Lambda_5^2} (\sigma^2 m^2 - \Lambda_5^2 \rho^2) . \quad (3.17)$$

Summarizing, the $|H^2|$ factor for the three cases is given by

$$|H^2| = \begin{cases} \frac{4}{3} \frac{|1-4\lambda|}{\rho^3 \kappa^2} \frac{m^2}{\Lambda_5^2} (\sigma^2 m^2 - \Lambda_5^2 \rho^2) , & \frac{|\Lambda_5|}{m} < \frac{\sigma}{\rho} \text{ for } dS_4 \text{ branes} , \\ 0 , & \frac{|\Lambda_5|}{m} = \frac{\sigma}{\rho} \text{ for flat branes} , \\ \frac{4}{3} \frac{|1-4\lambda|}{\rho^3 \kappa^2} \frac{m^2}{\Lambda_5^2} (\Lambda_5^2 \rho^2 - \sigma^2 m^2) , & \frac{|\Lambda_5|}{m} > \frac{\sigma}{\rho} \text{ for } AdS_4 \text{ branes} . \end{cases} \quad (3.18)$$

For positive values of Λ_5 we have the same solutions but for the opposite domains of λ and with positive sign for m^2 in (3.16). The solutions we found are similar to the solutions discussed in [32] for curved backgrounds.

IV. HIGHER EXTRINSIC CURVATURE OPERATORS

The action (2.3) receives naturally radiative corrections in the form of higher dimensional operators. The dimension of these operators depends on w if they involve $K_{\mu\nu}$, or are w -independent if they involve only $R^{(4)}$. It is reasonable to consider the theory only at energies that $(R^{(4)})^2$ terms are subdominant, since these are bound to introduce ghosts. For such energies, depending on the scaling w of the extra dimension, one could add higher powers of the extrinsic curvature in the action. Since $[(R^{(4)})^2] = 4$ and $[K_{ij}] = w$, we need to consider these powers of K that fulfill $nw < 4$. As an example, for $n = 3$ we obtain that a value of $w < \frac{4}{3}$ allows for cubic powers of the extrinsic curvature to be important at the energy region where the theory is still unitary.

Under these assumptions we are led to expand (2.3) by introducing the following terms

$$\Delta S = \int dy dx^4 \sqrt{-g} N \left(\frac{1}{\kappa^4} \frac{1}{\epsilon} (\alpha K^3 + \beta K_{\mu\nu} K^{\mu\nu} K + \gamma K_{\mu\nu} K^{\nu\rho} K_\rho^\mu) \right) , \quad (4.1)$$

where as before $[\kappa] = \frac{w-4}{2}$. The coupling ϵ scales as $[\epsilon] = 4$ and α, β, γ are dimensionless constants. The above action is split into three pieces $\Delta S = \Delta S_\alpha + \Delta S_\beta + \Delta S_\gamma$ which we are going to vary separately.

The variations of the above terms with respect to $g_{\mu\nu}$ give

$$\frac{1}{\sqrt{-g}} \frac{\delta \Delta S_\alpha}{\delta g_{\mu\nu}} = N \frac{1}{\kappa^4} \frac{1}{\epsilon} \alpha [N (-K^3 g^{\mu\nu} - 3K^2 K^{\mu\nu}) - \frac{3}{2} \partial_y (K^2 g^{\mu\nu}) - 3\nabla_\alpha (K^2 g^{(\mu\alpha} N^{\nu)}) + \frac{3}{2} N^\alpha \nabla_\alpha (K^2 g^{\mu\nu})] , \quad (4.2)$$

$$\frac{1}{\sqrt{-g}} \frac{\delta \Delta S_\beta}{\delta g_{\mu\nu}} = \frac{1}{\kappa^4} \frac{1}{\epsilon} \beta [-N (2K K^{\mu\sigma} K_\sigma^\nu + K^{\alpha\beta} K_{\alpha\beta} K^{\mu\nu} + K^2 K^{\mu\nu}) \quad (4.3)$$

$$\begin{aligned} & - (\partial_y (K K^{\mu\nu}) + \frac{1}{2} \partial_y (K^{\alpha\beta} K_{\alpha\beta} g^{\mu\nu}) + 2\nabla_\alpha (K^{\alpha(\mu} N^{\nu)} K) \\ & + \nabla_\alpha (g^{a(\mu} N^{\nu)} K^{\rho\sigma} K_{\rho\sigma}) - N^\alpha \nabla_\alpha (K K^{\mu\nu}) - \frac{1}{2} N^\alpha \nabla_\alpha (K^{\kappa l} K_{\kappa l} g^{\mu\nu})] , \\ \frac{1}{\sqrt{-g}} \frac{\delta \Delta S_\gamma}{\delta g_{\mu\nu}} &= \frac{1}{\kappa^4} \frac{1}{\epsilon} \gamma [N (\frac{1}{2} g^{\mu\nu} K_{\lambda\kappa} K^{\kappa\sigma} K_\sigma^\lambda - 3K_\lambda^\mu K^{\nu\sigma} K_\sigma^\lambda - \frac{3}{2} K K_\lambda^\mu K^{\nu\lambda}) \quad (4.4) \\ & - (\partial_y (\frac{3}{2} K_\sigma^\mu K^{\nu\sigma}) + 3\nabla_\alpha (K_\sigma^\alpha K^{(\mu\sigma} N^{\nu)}) - N^\alpha \nabla_\alpha (\frac{3}{2} K_\sigma^\mu K^{\nu\sigma}))] . \end{aligned}$$

The variations with respect to N_μ give

$$\frac{1}{\sqrt{-g}} \frac{\delta \Delta S_\alpha}{\delta N_\mu} = \frac{1}{\kappa^4} \frac{1}{\epsilon} \alpha (3 \nabla_\nu (K^2 g^{\mu\nu})) , \quad (4.5)$$

$$\frac{1}{\sqrt{-g}} \frac{\delta \Delta S_\beta}{\delta N_\mu} = \frac{1}{\kappa^4} \frac{1}{\epsilon} \beta (\nabla_\nu (2K^{\mu\nu} K + K^{\rho\sigma} K_{\rho\sigma} g^{\mu\nu})) , \quad (4.6)$$

$$\frac{1}{\sqrt{-g}} \frac{\delta \Delta S_\gamma}{\delta N_\mu} = \frac{1}{\kappa^4} \frac{1}{\epsilon} \gamma (\nabla_\nu (3 K_\sigma^\mu K_\sigma^\nu)) . \quad (4.7)$$

Finally the variation with respect to N is

$$\frac{1}{\sqrt{-g}} \frac{\delta \Delta S}{\delta N} = \frac{1}{\kappa^4} \frac{1}{\epsilon} (-2) (\alpha K^3 + \beta K_{\mu\nu} K^{\mu\nu} K + \gamma K_{\mu\nu} K^{\nu\sigma} K_\sigma^\mu) . \quad (4.8)$$

As before we need to examine the junction conditions across the brane. Integrating (4.11) and focusing on the distributional part, we see that the junction condition reads

$$\left[\frac{2}{\kappa^2} \pi^{\mu\nu} + \frac{1}{\kappa^4} \frac{1}{\epsilon} \left(-\alpha \frac{3}{2} K^2 g^{\mu\nu} - \beta \left(K K^{\mu\nu} + \frac{1}{2} K^{\sigma\kappa} K_{\sigma\kappa} g^{\mu\nu} \right) - \gamma \frac{3}{2} K^\mu_\sigma K^{\nu\sigma} \right) \right]_-^+ = \frac{1}{2} \sigma g^{\mu\nu} . \quad (4.9)$$

Taking into account the above variations and comparing with the equations of motion of the quadratic terms of the extrinsic curvature, we see that in the case where $\gamma = -4\beta - 16\alpha$ we get exactly the same results both for flat and curved branes. This is because in this limit (4.1) vanishes. In the case of flat branes, even deviating from this limit, we obtain solutions of the same form with redefined constants.

A. Flat Branes

For the case of flat branes we consider the ansatz (3.1). Then (2.5) along side with (4.8) becomes

$$-\rho \Lambda_5 + \frac{8(1-4\lambda)(f'(y))^2}{\kappa^2} - \frac{8(16\alpha+4\beta+\gamma)}{\epsilon \kappa^4} (f'(y))^3 = 0 . \quad (4.10)$$

Equation (2.6) together with equations (4.5), (4.6) and (4.7) are satisfied identically. Equation (3.3) alongside with equations (4.2), (4.2) and (4.4) give the following expression

$$\begin{aligned} & -\frac{\rho}{2} g^{\mu\nu} \Lambda_5 N + \frac{2}{\kappa^2} [\partial_y \pi^{\mu\nu} + N K \pi^{\mu\nu} + 2N K^{\sigma\mu} \pi^\nu_\sigma] - \frac{1}{2} g^{\mu\nu} \left(\frac{2}{\kappa^2} (K_{\mu\nu} K^{\mu\nu} - \lambda K^2) \right) - \frac{1}{2} \sigma g^{\mu\nu} \delta(y) \\ & + \frac{1}{\kappa^4} \frac{1}{\epsilon} \left[\alpha \left(-K^3 g^{\mu\nu} - 3K^2 K^{\mu\nu} - \frac{3}{2} \partial_y (K^2 g^{\mu\nu}) \right) \right. \\ & + \beta \left(-2K K^{\mu\sigma} K^\nu_\sigma - K^{\sigma\kappa} K_{\sigma\kappa} K^{\mu\nu} - \partial_y (K K^{\mu\nu}) - \frac{1}{2} \partial_y (K^{\sigma\kappa} K_{\sigma\kappa} g^{\mu\nu}) \right) \\ & \left. + \gamma \left(\frac{1}{2} g^{\mu\nu} K_{\sigma\kappa} K^{\kappa\lambda} K^\sigma_\lambda - 3K^\mu_\lambda K^{\nu\sigma} K^\sigma_\sigma - \frac{3}{2} K K^\mu_\rho K^{\nu\rho} - \frac{3}{2} \partial_y (K^\mu_\sigma K^{\nu\sigma}) \right) \right] = 0 , \end{aligned} \quad (4.11)$$

from which we get

$$\begin{aligned} & \epsilon \kappa^4 \Lambda_5 \rho + 8(f'(y))^2 (\epsilon \kappa^2 (-1+4\lambda) + (16\alpha+4\beta+\gamma) f'(y)) + \epsilon \kappa^4 \sigma \delta(y) \\ & + 2(2\epsilon \kappa^2 (-1+4\lambda) + 3(16\alpha+4\beta+\gamma) f'(y)) f''(y) = 0 . \end{aligned} \quad (4.12)$$

Depending on the value of the expression $(16\alpha+4\beta+\gamma)$ we can distinguish the following cases:

1. Case 1 $(16\alpha+4\beta+\gamma) = 0$

It is clear that in the case where the dimensionless constants satisfy $(16\alpha+4\beta+\gamma) = 0$, we end with the same results as in the case of the K^2 terms.

2. Case 2 $(16\alpha+4\beta+\gamma) \neq 0$

When the above constraint is relaxed, solving (4.10) for $(f'(y))^3$ and substituting it to (4.12) we get

$$2(2\epsilon \kappa^2 (-1+4\lambda) + 3(16\alpha+4\beta+\gamma) f'(y)) f''(y) = 0 . \quad (4.13)$$

Now if we have that

$$2\epsilon \kappa^2 (-1+4\lambda) + 3(16\alpha+4\beta+\gamma) f'(y) = 0 , \quad (4.14)$$

then

$$f(y) = \frac{2|y|\epsilon \kappa^2 (1-4\lambda)}{3(16\alpha+4\beta+\gamma)} , \quad (4.15)$$

and

$$\Lambda_5 = -\frac{32\epsilon^2\kappa^2(-1+4\lambda)^3}{27(16\alpha+4\beta+\gamma)^2\rho} . \quad (4.16)$$

We note here that, contrary to the K^2 terms, the behaviour of the warp factor $f(y)$, if it is growing or decaying, depends on the value of $(1-4\lambda)/(16\alpha+4\beta+\gamma)$. Furthermore, in this case of the K^3 terms the sign of Λ_5 , depends on the value of λ .

If on the other hand $f''(y) = 0$, *i.e.*

$$f(y) = Ay + B, \quad (4.17)$$

we have that

$$\Lambda_5 = -\frac{8A^2(\epsilon\kappa^2(-1+4\lambda) + (16\alpha+4\beta+\gamma)A)}{\epsilon\kappa^4\rho} . \quad (4.18)$$

At the conformal point where $\lambda \rightarrow 1/4$, we have that $f'(y)f''(y) = 0$, so either $f(y) = \text{const}$ and $\Lambda_5 = 0$, or $f(y) = Ay + B$ and $\Lambda_5 = -8(16\alpha+4\beta+\gamma)/\epsilon\kappa^4\rho$. Again we see that the behaviour of the warp factor and the cosmological constant depend on the choice of parameters.

Applying in the above the junction conditions (4.9), we get

$$\epsilon\kappa^4\sigma\delta(y) + 4\epsilon\kappa^2(-1+4\lambda)[f'(y)]_-^+ + 6(16\alpha+4\beta+\gamma)[(f'(y))^2]_-^+ = 0 . \quad (4.19)$$

Due to the Z_2 symmetry, the quadratic term vanishes, resulting to the same junction equations as in the case of K^2 terms. Substituting the solution for $f(y)$ ((4.15)) we get

$$\sigma = \frac{16}{3}\epsilon\frac{(1-4\lambda)^2}{(16\alpha+4\beta+\gamma)} , \quad (4.20)$$

and for the solution (4.17)

$$\sigma = -\frac{8}{\kappa^2}(-1+4\lambda)A . \quad (4.21)$$

Again in the conformal limit the brane is tensionless. In order to get significant changes to the junction conditions we have to move to K^4 terms, since these terms will produce terms $(f'(y))^3$ which are even.

V. SCALAR PERTURBATIONS

In this section we study the scalar sector of perturbations of the theory that has up to quadratic terms of the extrinsic curvature. For that purpose, we will use the flat vacua of the theory, analysed in section IIIA. We consider the following metric ansatz ²,

$$N = e^{\alpha(x^\nu, y)}, \quad N_\mu = \partial_\mu \beta(x^\nu, y), \quad g_{\mu\nu} = e^{2(f(y) + \zeta(x^\rho, y))} \eta_{\mu\nu} , \quad (5.1)$$

which differs from the most general scalar perturbation possibly by a perturbation of $g_{\mu\nu}$ of the form $2\partial_\mu\partial_\nu E$, which however can be gauged away (see [15]).

Using the above ansatz, we compute in Appendix A the various invariants appearing in the action. Inserting them back in the action and keeping terms up to quadratic order in perturbations, we obtain the following quadratic bulk action

² Note that the lapse function N depends also on the 4-dimensional spacetime coordinates. Therefore, in general, terms depending on derivatives of N are allowed in the action [14]. This would significantly increase the number of terms in the action and to keep the analysis tractable we assume their couplings to be small so they can be ignored.

$$\begin{aligned}
S = & \int dx^4 dy e^{2f} [\rho \left(3 \left((\partial \zeta)^2 - \alpha \square^{(4)} \zeta \right) - e^{2f} \left(1 + 4\zeta + \alpha + 4\alpha\zeta + \frac{\alpha^2}{2} + 8\zeta^2 \right) \Lambda_5 \right) \\
& - \frac{2}{\kappa^2} (4e^{2f} (1 - 4\lambda) ((\partial_y f)^2 + 2\partial_y f \partial_y \zeta + (\partial_y \zeta)^2 - \alpha (\partial_y f)^2 - 2\alpha \partial_y f \partial_y \zeta + \frac{\alpha^2}{2} (\partial_y f)^2 \\
& + 4\zeta (\partial_y f)^2 + 8\zeta \partial_y f \partial_y \zeta - 4\alpha\zeta (\partial_y f)^2 + 8\zeta^2 (\partial_y f)^2) \\
& + 2(1 - 4\lambda) \partial_y f \alpha \square^{(4)} \beta - 4(1 - 4\lambda) \partial_y f \zeta \square^{(4)} \beta - 2(1 - 4\lambda) \partial_y \zeta \square^{(4)} \beta \\
& - 4(1 - 4\lambda) \partial_y f \partial^\nu \zeta \partial_\nu \beta + e^{-2f} (1 - \lambda) \left(\square^{(4)} \beta \right)^2] .
\end{aligned} \tag{5.2}$$

This action has one non-dynamical degree of freedom, α . Varying the action with respect to α , produces a constraint, to be imposed to the system, which reads

$$\begin{aligned}
3\rho \square^{(4)} \zeta + \frac{4}{\kappa^2} (1 - 4\lambda) \partial_y f \square^{(4)} \beta = & \frac{8}{\kappa^2} e^{2f} (1 - 4\lambda) \left((\partial_y f)^2 + 2\partial_y f \partial_y \zeta - \alpha (\partial_y f)^2 + 4\zeta (\partial_y f)^2 \right) \\
& - e^{2f} \rho \Lambda_5 (1 + 4\zeta + \alpha) .
\end{aligned} \tag{5.3}$$

Using equation (5.3) we can eliminate α in favor of the ζ and β in the action (5.2) and then we obtain

$$\begin{aligned}
S = & \int dx^4 dy e^{2f} [24\partial_y f (\partial_y \zeta) \square^{(4)} \zeta \frac{(-1 + 4\lambda)}{\Lambda_5 \kappa^2} + 16\partial_y f \partial_y \zeta (-1 + 4\lambda) e^{2f} \frac{(1 + 4\zeta)}{\kappa^2} \\
& + 3\rho (\partial \zeta)^2 + \frac{9\rho}{4\Lambda_5} e^{-2f} \left(\square^{(4)} \zeta \right)^2 - \frac{3}{2\kappa^2} e^{-2f} \left(\square^{(4)} \beta \right)^2 - 6(\partial_y f) e^{-2f} \frac{(-1 + 4\lambda)}{\kappa^2 \Lambda_5} \square^{(4)} \beta \square^{(4)} \zeta - 2e^{2f} (1 + 4\zeta + 8\zeta^2) \Lambda_5 \rho] \\
& - \int dx^4 dy e^{2f} (1 + 4\zeta + 8\zeta^2) e^{2f} \sigma \delta(y) .
\end{aligned} \tag{5.4}$$

The last term in the above action is the brane boundary term appearing in (2.3). The above action, as it is explained in the Appentix A, after appropriate partial integrations (while assuming appropriate boundary conditions), can be brought to the final form

$$\begin{aligned}
S = & \int dx^4 dy e^{2f} \left[\frac{9\rho}{4\Lambda_5} e^{-2f} \left(\square^{(4)} \zeta \right)^2 - \frac{3}{2\kappa^2} e^{-2f} \left(\square^{(4)} \beta \right)^2 - 6(\partial_y f) e^{-2f} \frac{(-1 + 4\lambda)}{\kappa^2 \Lambda_5} \square^{(4)} \beta \square^{(4)} \zeta \right. \\
& \left. + \frac{6\sqrt{2}}{\kappa} \frac{(1 - 4\lambda)}{\Lambda_5} \sqrt{\frac{\Lambda_5 \rho}{1 - 4\lambda}} (\partial \zeta)^2 \delta(y) \right] .
\end{aligned} \tag{5.5}$$

This is the main result of our work. The terms in the first line are higher derivative terms for the two dynamical degrees of freedom ζ and β , which have kinetic mixing between them. As a check of the correctness of the above action, one can vary it with respect to β . Then we obtain the same result with the variation of (5.2) which reads

$$(1 - \lambda) e^{-2f} \square^{(4)} \beta = (1 - 4\lambda) (\partial_y \zeta - \partial_y f \alpha) . \tag{5.6}$$

The equivalence can be seen by substituting (5.3) in (5.6). Similarly, the variation with respect to ζ gives the same result once the constraint is taken into account.

The action (5.5) has certain characteristics. First, all the bulk terms involve four derivatives of brane coordinates, and certain terms have four time derivatives. Secondly, the brane term is a ghost-like kinetic term for ζ . Once the wave-functions for the two dynamical modes have been substituted, the four dimensional action will be consisted from the ghost kinetic term for ζ , plus higher derivative terms. These terms will appear in the action multiplied with different scales. However, whatever the hierarchy of these scales may be, there will always be some ghost problem in the spectrum: either from the quadratic kinetic term, or from the higher derivative terms.

These modes will be present even after the restoration of the five-dimensional diffeomorphism symmetry, *i.e.* when $\lambda \rightarrow 1$. This last characteristic, is very similar to what happens in massive gravity, namely the vDVZ discontinuity [26, 27]. In that case, similarly the longitudinal mode of the massive graviton does not decouple in the limit of vanishing Pauli-Fierz mass. In our example, we have some even worse result, since the remaining modes have ghost behaviour. These problems are probably due to the explicit breaking of the diffeomorphism invariance in the theory. The theory may not appear problematic at the level of background solutions, but nevertheless these problems reveal themselves once the theory is perturbed.

VI. CONCLUSIONS

We investigated the consequences of an assumption that the fifth extra dimension scales differently than the other four dimensions. To achieve this we considered a five-dimensional theory with a cosmological constant and a three brane embedded in it. In this theory, the full five-dimensional diffeomorphism group is explicitly broken down to its foliation-preserving four-dimensional subgroup. The foliation we used involves an extra space dimension and therefore the four-dimensional Lorentz invariance is intact. Because of the different scaling of the extra spacial dimension, higher powers of the extrinsic curvature of the four-dimensional hypersurfaces are allowed in the theory up to the energies that ghost-bearing higher order dimensional operators appear.

We made a systematic study of the local solutions of this theory. For maximally symmetric backgrounds and up to second order in extrinsic curvature we found all solutions for flat and curved branes. These solutions are similar to the previously obtained solutions for flat and curved branes, they are however characterized by a parameter λ that expresses the breaking of the five-dimensional diffeomorphism invariance. We also obtained solutions in the flat brane limit by including higher order in extrinsic curvature terms. These solutions except their dependence on λ also depend on the coefficients by which the higher order extrinsic curvature terms enter in the action.

Having explicitly broken the Lorentz invariance in five dimensions, we looked for possible effects on the four dimensional spacetime. We performed a scalar perturbations analysis of the theory for up to quadratic terms in the extrinsic curvature. We found that, as a result of the breaking of the diffeomorphism invariance, two dynamical scalar degrees of freedom, appear. Both of them are characterized by the fact that they come as higher derivative corrections to the action. Moreover one of these modes appears with a ghost-like brane kinetic term. These pathologies can be attributed to the explicit breaking of diffeomorphism invariance along the extra space dimension.

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Appendix A: Technical Details for Scalar perturbations

In this Appendix we give the technical details for deriving the action (5.5) in section V. We start with the generalize action (2.3) ignoring for the moment the boundary brane term

$$S = \int N \sqrt{-g} dy dx^4 \left[\frac{\rho}{2} \left(R^{(4)} - 2\Lambda_5 \right) - \frac{2}{\kappa^2} (K_{\mu\nu} K^{\mu\nu} - \lambda K^2) \right]. \quad (A1)$$

We perturb the above action, in the flat brane limit discussed in section IIIA, using the perturbation ansatz (5.1) for the scalar perturbations. The extrinsic curvature and its trace is given by

$$K^{\mu\nu} = e^{-4(f+\zeta+\frac{\alpha}{2})} \left[e^{2(f+\zeta)} \partial_y (f + \zeta) \eta^{\mu\nu} - \partial^\mu \partial^\nu \beta + \partial^\mu \zeta \partial^\nu \beta + \partial^\nu \zeta \partial^\mu \beta - \eta^{\mu\nu} \partial^\lambda \zeta \partial_\lambda \beta \right], \quad (A2)$$

$$K = e^{-2(f+\zeta+\frac{\alpha}{2})} \left[4 e^{2(f+\zeta)} \partial_y (f + \zeta) - \square^{(4)} \beta - 2 \partial^\lambda \zeta \partial_\lambda \beta \right], \quad (A3)$$

where $\square^{(4)} = \eta^{\mu\nu} \partial_\mu \partial_\nu$, while the Ricci scalar is

$$R = -6 e^{-2(f+\zeta)} \left(\square^{(4)} \zeta + (\partial \zeta)^2 \right). \quad (A4)$$

Collecting the above terms, the perturbed action, up to quadratic order, reads

$$\begin{aligned} S = & \int dx^4 dy e^{2f} \left[\rho \left(3 \left((\partial \zeta)^2 - \alpha \square^{(4)} \zeta \right) - e^{2f} \left(1 + 4\zeta + \alpha + 4\alpha\zeta + \frac{\alpha^2}{2} + 8\zeta^2 \right) \Lambda_5 \right) \right. \\ & - \frac{2}{\kappa^2} (4e^{2f} (1 - 4\lambda) ((\partial_y f)^2 + 2\partial_y f \partial_y \zeta + (\partial_y \zeta)^2 - \alpha (\partial_y f)^2 - 2\alpha \partial_y f \partial_y \zeta + \frac{\alpha^2}{2} (\partial_y f)^2 \\ & + 4\zeta (\partial_y f)^2 + 8\zeta \partial_y f \partial_y \zeta - 4\alpha \zeta (\partial_y f)^2 + 8\zeta^2 (\partial_y f)^2) \\ & + 2(1 - 4\lambda) \partial_y f \alpha \square^{(4)} \beta - 4(1 - 4\lambda) \partial_y f \zeta \square^{(4)} \beta - 2(1 - 4\lambda) \partial_y \zeta \square^{(4)} \beta \\ & \left. - 4(1 - 4\lambda) \partial_y f \partial^\nu \zeta \partial_\nu \beta + e^{-2f} (1 - \lambda) \left(\square^{(4)} \beta \right)^2 \right]. \end{aligned} \quad (A5)$$

The above action contains one non-dynamical field, namely α . Varying the action with respect to α , we get a constraint,

$$3\rho\Box^{(4)}\zeta + \frac{4}{\kappa^2}(1-4\lambda)\partial_y f\Box^{(4)}\beta = \frac{8}{\kappa^2}e^{2f}(1-4\lambda)\left((\partial_y f)^2 + 2\partial_y f\partial_y\zeta - \alpha(\partial_y f)^2 + 4\zeta(\partial_y f)^2\right) - e^{2f}\rho\Lambda_5(1+4\zeta+\alpha), \quad (\text{A6})$$

while the equation for β is the following

$$(1-\lambda)e^{-2f}\Box^{(4)}\beta = (1-4\lambda)(\partial_y\zeta - \partial_y f\alpha). \quad (\text{A7})$$

Using (A6) we can eliminate α from the action (A5) and get the following expression for the action where we have two dynamical fields appearing in the action, ζ and β

$$\begin{aligned} S = & \int dx^4 dy e^{2f} \left[24\partial_y f(\partial_y\zeta)\Box^{(4)}\zeta \frac{(-1+4\lambda)}{\Lambda_5\kappa^2} + 16\partial_y f\partial_y\zeta(-1+4\lambda)e^{2f}\frac{(1+4\zeta)}{\kappa^2} \right. \\ & + 3\rho(\partial\zeta)^2 + \frac{9\rho}{4\Lambda_5}e^{-2f}(\Box\zeta)^2 - \frac{3}{2\kappa^2}e^{-2f}(\Box\beta)^2 - 6(\partial_y f)e^{-2f}\frac{(-1+4\lambda)}{\kappa^2\Lambda_5}\Box\beta\Box\zeta - 2e^{2f}(1+4\zeta+8\zeta^2)\Lambda_5\rho \\ & \left. - \int dx^4 dy e^{2f}(1+4\zeta+8\zeta^2)e^{2f}\sigma\delta(y) \right]. \end{aligned} \quad (\text{A8})$$

Note that we have restated the boundary brane term of the original action which we had neglected so far. This action can be simplified as follows. Consider the following terms

$$\int dx^4 dy e^{2f} \left[16\partial_y f\partial_y\zeta(-1+4\lambda)e^{2f}\frac{1+4\zeta}{\kappa^2} - 2e^{2f}(1+4\zeta+8\zeta^2)\Lambda_5\rho \right], \quad (\text{A9})$$

and perform a partial integration with respect to the extra dimension using (3.4). Then these terms are equal to

$$- \int dx^4 dy e^{4f} 2\sqrt{2}\frac{(1-4\lambda)}{\kappa}\sqrt{\frac{\Lambda_5\rho}{1-4\lambda}}(1+4\zeta+8\zeta^2)\delta(y),$$

which is exactly the same as the contribution of the boundary term appearing in (A8) with an opposite sign, after having used the value of the brane tension (3.6). From the remaining terms of the action (A8) consider the following terms

$$\int dx^4 dy e^{2f} \left[24\partial_y f(\partial_y\zeta)\Box^{(4)}\zeta \frac{(-1+4\lambda)}{\Lambda_5\kappa^2} + 3\rho(\partial\zeta)^2 \right]. \quad (\text{A10})$$

Observing that

$$\int dx^4 dy e^{2f}\partial_y[\partial_\mu\zeta\partial^\mu\zeta] = \int dx^4 dy e^{2f}(-2(\partial_y\zeta)\Box^{(4)}\zeta), \quad (\text{A11})$$

where we have performed a partial integration with respect to the brane coordinates and assumed appropriate boundary conditions, the above terms become

$$\begin{aligned} & \int dx^4 dy \left[3\rho(\partial\zeta)^2 e^{2f} + 12(-2(\partial_y\zeta)\Box^{(4)}\zeta)\partial_y f e^{2f}\frac{(1-4\lambda)}{\Lambda_5\kappa^2} \right] = \\ & \int dx^4 dy \left[3\rho(\partial\zeta)^2 e^{2f} + 12\partial_y((\partial\zeta)^2)\partial_y f e^{2f}\frac{(1-4\lambda)}{\Lambda_5\kappa^2} \right] = \\ & \int dx^4 dy e^{2f}\frac{6\sqrt{2}}{\kappa}\frac{(1-4\lambda)}{\Lambda_5}\sqrt{\frac{\Lambda_5\rho}{1-4\lambda}}(\partial\zeta)^2\delta(y). \end{aligned} \quad (\text{A12})$$

Using (A12) the final result of the perturbed action (2.3) under the scalar perturbations of the form (5.1) is

$$\begin{aligned} S = & \int dx^4 dy e^{2f} \left[\frac{9\rho}{4\Lambda_5}e^{-2f}(\Box\zeta)^2 - \frac{3}{2\kappa^2}e^{-2f}(\Box\beta)^2 - 6(\partial_y f)e^{-2f}\frac{(-1+4\lambda)}{\kappa^2\Lambda_5}\Box\beta\Box\zeta \right. \\ & \left. + \frac{6\sqrt{2}}{\kappa}\frac{(1-4\lambda)}{\Lambda_5}\sqrt{\frac{\Lambda_5\rho}{1-4\lambda}}(\partial\zeta)^2\delta(y) \right]. \end{aligned} \quad (\text{A13})$$

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